

# Challenging Logical Monism

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## Author Bio

Aurna Mukherjee is in 12th grade at Liberal Arts and Science Academy, a highly-ranked magnet high school in Austin, Texas. She became interested in Philosophy and Logic in 10th grade and received a scholarship to participate in the Cambridge Center for International Research in 2023. She has been working with Dr. Owen Griffiths from Cambridge University as part of her research project on logical monism, and recently presented her paper at an undergraduate conference (Perspectives About Truth 2).

## Abstract

Logic is loosely regarded as a key factor that drives our decisions. However, logic is actually separated into different systems, such as intuitionistic logic and classical logic. These systems can be explained by different theories, such as logical monism and logical pluralism. This paper aims to challenge logical monism, which posits that only a single logical system adheres to the principles of validity. It explains this on the basis of different systems held as equally strong in terms of their ability to establish definitive conclusions, and adhere to systems of validity relative to each system.

*Keywords:* Logical Monism, Validity, Logic, Logical systems, Reasoning, Rules of Validity, Logical pluralism, Intuitionistic logic, Classical logic.

## Introduction

What is logic? In everyday discourse, it is held as a set of rules to reach a certain outcome. For example, in order to have a chance at winning the lottery, it would be logical to buy more tickets than others to have a better chance at this. In philosophical terms however, it is accepted as the study of arguments made with correct reasoning. The formal definition offers some clarity into what makes up “correct reasoning.” When considering the word formally, it is the study of reasoning conducted by rules of validity. This raises the question of what validity is. Roughly, an argument is considered valid when there is no case with true premises or propositions that serve as the basis of an argument, which leads to a false conclusion.

More broadly, a conclusion must follow from its premises (Shapiro & Kissel, 2018). It is not entirely clear what “follows from” means in this context, or when the case of true premises and a false conclusion arises. However, there are several different logical systems, each associated with individual rules of validity, where this case can occur. Two of these theories are classical logic and intuitionistic logic, which are both associated with individual definitions of validity (Moschovakis, 2023). In classical logic, any conclusion in the form of  $P$  or not- $P$  is valid, regardless of what the premise is. In this case,  $P$  or not- $P$  means that something must either be true ( $P$ ) or false (not- $P$ ) and having one of the two outcomes is regarded as true in classical logic (Plantin, 2021). Intuitionistic logic does not follow this notion. Which one is correct? Is it possible that both intuitionistic and classical logic can be? These questions are addressed by the notions of logical monism and logical pluralism.

To explain the difference in validity that several logical systems can have, monists hold that only one logical system suffices as valid, which I will argue against (Clark-Doane & Peacocke, 2022). Pluralists have a separate interpretation, maintaining that all logical systems are considered “equally strong” under their different definitions of validity (Beall & Restall, 2000). The meaning of “equally strong” is elaborated later. The definition of validity becomes relative to the logical system, so this overarching meaning of validity is not enough.

## Validity in Logical Systems

As mentioned before, the definition of validity varies when considering two logical systems, classical logic and intuitionistic logic. Classical logic is often held as the one true logic that monists follow, meaning that other logical systems, such as intuitionistic logic, are not correct (Shapiro & Kissel, 2018). This is because the rules of validity are relative to each system. As Stanford Encyclopedia of Philosophy author Joan Moschovakis describes, intuitionistic logic excludes two laws that are established in classical logic, making it a subset of the classical logic system (2023). Because of this, certain arguments can be considered valid in classical logic. Two of the laws used to establish validity are not assumptions in intuitionistic logic, so these same arguments can be invalid in intuitionistic logic. Beyond various logical systems, the traditional definition of validity aligns in mathematical practice. In this lens, the conclusion of an equation must follow from the mathematical operations being utilized. This is similar to the formal definition of validity in the context of arguments, where the conclusion must follow from its premises (Shapiro & Kissel, 2018). Because of these differences, if we approach the question of whether logical monism or logical pluralism is accurate from the perspective of logical monism being an incomplete view of validity, we gain a new understanding. I argue that logical pluralism is a more accurate way to understand why different types of logic may lend themselves to various outcomes.

## Classical Logic

As mentioned previously, logical monists regard classical logic as the most accepted system. It arose from Aristotle, making it one of the oldest and most well-established logical systems that exist (Shapiro & Kissel, 2018). In the view of a logical monist, other logical systems are not valid. Camillo Fiore and Lucas Rosenblatt describe the definition characteristics of the classical logic system in a separate paper (Fiore & Rosenblatt, 2023). Classical first order logic uses individual contexts and variables to denote specific objects and express generality – it involves a distributive lattice with logical operators, meaning that distributivity laws in mathematics are satisfied. For example,  $p \wedge (q \vee r)$  can be distributed as  $(p \wedge q) \vee (p \wedge r)$ . These basic distributive laws

are intuitive and relevant to several logical systems. It is the additional principles of classical logic that differentiate it from some other logic systems, such as intuitionistic logic. As Christian Plantin asserts in *An Art of Thinking*, it has certain foundational principles, such as non-contradiction, the excluded middle, and the identity “ $a=a$ ” (2021). With these principles assumed to be true, they shape the validity of arguments under classical logic. Non-contradiction is the principle that a proposition cannot be both true and false, as this would be contradictory. Only one of these two options makes logical sense. The excluded middle asserts either  $P$  or not- $P$ . If  $P$  is true, then not- $P$  is false. If  $P$  is false, then not- $P$  is true. This establishes that the proposition must either be true or false. The identity “ $a=a$ ” means that each variable must be equal to itself. The concept of double negation is present, where  $\neg\neg P \equiv P$ .  $\neg$  means not in this context. If  $P$  is false,  $\neg P$  is true, and  $\neg\neg P$  is false. If  $P$  is true,  $\neg P$  is false, and  $\neg\neg P$  is true, and this is the only other outcome possible, proving this conjecture (Plantin, 2021). In more simplistic terms, let us consider ice cream. If someone says they do not-not have ice cream, classical logic would simplify this to they do have ice cream. With these assumptions, classical logic can establish validity with these laws in place, whereas other logical systems cannot. In the example established above, the logical system of intuitionistic logic would not consider the statement valid. Along with this, the use of conditionals can establish false premises under classical logic.

For example, if  $P$  and not- $P$  is a premise, this is deemed false under classical logic (Shapiro & Kissel, 2018). Consider ice cream again - if someone says that they both have and do not have ice cream, this would be held false. While seemingly counterintuitive, the definition of validity previously mentioned asserts that any argument with false premises is valid, so in the case of both having and not having ice cream, the argument would have to be valid with the false premise. This logical system provides a basis for deductive reasoning, which is the process of progressing from general ideas to specific conclusions, consistent with the definition of validity, where the conclusion follows from the premises of an argument (Bhandari, 2022). Under the concept of logical monism, this is a logical system that holds relevance and is considered valid, so only propositions established in classical logic should remain valid.

## Intuitionistic Logic

A clear counterargument to the position of logical monism holding classical logic to be the “one true logic” is intuitionistic logic, as only one of the two can be true (Bezhanishvili & Jongh, 2010). Nick Bezhanishvili and Dick De Jongh describe the principles of the intuitionistic logic system. It is a separate type of logical system that builds its arguments as a constructive proof. It is the foundation of advanced mathematical systems, which allows for quick computing of complex integrals and derivatives. It is normative, as it uses a standard value of an argument, rather than enforcing any assumptions, as classical logic would (Bezhanishvili & Jongh, 2010). In order to use this standard value, it centers around building upon assumptions, rather than destroying constructivity (Moschovakis, 2023). What does constructivity mean? It refers to mathematical objects at face value and derives conclusions based on this (Mints, 2020). This means that laws which assume one of two truth values, rather than considering all possible outcomes, are rejected. The law of the excluded middle and the law of double negation elimination are not included in intuitionistic logic (Mints, 2020). Let us consider double negation.  $\neg\neg P \equiv P$  would result in a false outcome, as double negation is not established in this logical system. In classical logic, the double negation operator would lead to either “true” or “false,” but this is not assumed in intuitionistic logic. Under logical monism, the explained reason for this is that there is only one type of logical system that is valid.

## Equally Strong

However, this seems highly unlikely, considering that other types of logic do exist, with the two described being intuitionistic logic and classical logic. Both are considered “equally strong.” By the phrase “equally strong,” I assert that we must look to a view that reaches a conclusion valid relative to its logical system. Author Boaz Schuman presents the Clean Divide View in the context of a separate logical system called Buridan logic, where there are two versions, one being the stronger and another being the weaker (Schuman, 2021). The stronger view provides a clear interpretation of the system, while the other asserts that an outcome is “probably technically false,” meaning it does not provide full insight into how

Buridan logic works. More generally, I interpret this principle to mean that within two views, if one reaches a definitive conclusion and demonstrates a reasonable function of its logical system, whereas the other does not, the former is the “stronger” logical system.

Suppose that there are two situations: in one logical system, conclusion B follows from premise A, yet in the other, the result of premise A is inconclusive. We can maintain that the former logical system is “stronger” than the other as there is certainty in the conclusion. In this situation, the systems are not “equally strong.” However, in the case of intuitionistic logic and classical logic, both are equally capable of this, meaning that both are equally strong. Classical logicians may refute this by asserting that classical logic can establish validity in places where intuitionistic logic cannot, and therefore, the system is more accurate. For example, consider the conclusion that I have or do not have ice cream. In addition, intuitionistic logic is a subset of classical logic, meaning that all conjectures proven in intuitionistic logic can also be established in classical logic (Moschovakis, 2023). The opposite is not true, which is why the law of double negation and excluded middle have different truth values in both systems. An easy way to understand this is considering squares and rectangles: squares are a subset of rectangles, yet not all rectangles are squares. The best explanation for the difference in validity between intuitionistic logic and classical logic is that the truth of an expression is relative to the logical system, so asserting one single type of logic does not account for these differences. Authors Camillo Fiore and Lucas Rosenblatt describe this principle as logical faithfulness: the theories being used should correspond to the logical system at hand (2023). Therefore, we can argue that there are multiple logical systems outside of classical logic that are just as important, and simply fall under different notions of validity.

## Goldbach Disjunction

Logical pluralism is not only applicable to formal arguments but also in mathematical practice. Critics assert that logical monism is true by considering the Goldbach Disjunction (Clark-Doane & Peacocke, 2022). The Goldbach conjecture is the claim that every natural number greater than two is the sum of two prime numbers. The Goldbach Disjunction

is the disjunction of the Goldbach conjecture and its negation. Classical logicians assert that the disjunction is true, by assuming that either the conjecture or its negation must be true (Clark-Doane & Peacocke, 2022). This is consistent with the law of the excluded middle, where either P or not-P is true (Plantin, 2021). This is met with disagreement from intuitionists, who argue that the disjunction cannot be established, as neither the conjecture nor its negation can be proven, so neither outcome can be clearly established (Panu, 2013). Both situations need to be constructed and proved, for intuitionists to treat the disjunction as true. Logical monism supports classical logic, meaning that the notion of intuitionism is disregarded altogether. In contrast, logical pluralism supports both theories. They are simply different logical systems, and attempting to compare the two which are made of different rules is a futile task. Even though in one system, the disjunction is considered valid, it is reasonable that in another it is not, because validity is relative to the logical system at hand.

## Counterargument

Logical monists will argue that certain arguments are valid on their own, and classical logic is the only system that satisfies these arguments. On its own, there is no case in which the conclusion of P or not-P is false, so the argument is valid – classical logic correctly identifies this argument to be valid. Intuitionistic logic cannot establish this, although it seems obvious. However, this argument is rather limited. Arguments on their own cannot objectively be valid or not valid; they are valid relative to the logical system they belong to. Multiple logic systems have independent rules of validity, so classical logic is valid in relation to its system, and intuitionistic logic is valid in relation to its system. The counter argument being made aligns with the rules of validity in the classical logic system, so it fails to look beyond this system. Therefore, it is imprecise to assert that the rules of classical logic are arbitrarily true.

## Conclusion

Logical monism is described as a widely accepted logical system. However, it does not incorporate the idea that there are various rules of validity relative to other logical systems, all of which are equally strong. Intuitionistic logic and classical logic are two such logical systems that have different rules of validity, but are equally strong relatively. A way to understand this is through logical pluralism, which holds that multiple logical systems exist with different rules of validity, which are equally strong, so assuming only one type of logical system that suffices is a feeble task. Based on these findings, it is clear that logical monism does not offer a realistic interpretation of the various logical systems that are existent. Logical pluralism is the best alternative to consider.

## Works Cited

- Beall, J., & Restall, G. (2000). Logical pluralism. *Australasian Journal of Philosophy*, 78(4), 475–493. <https://doi.org/10.1080/00048400012349751>
- Bezhanishvili, N., & De Jongh, D. (n.d.). Intuitionistic Logic. <https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.148.8538&rep=rep1&type=pdf>
- Bhandari, P. (2022, January 20). An easy introduction to deductive reasoning. Scribbr. <https://www.scribbr.com/methodology/deductive-reasoning/>
- Constructive logic - Encyclopedia of Mathematics. (n.d.). [encyclopediaofmath.org](https://encyclopediaofmath.org/wiki/Constructive_logic). [https://encyclopediaofmath.org/wiki/Constructive\\_logic](https://encyclopediaofmath.org/wiki/Constructive_logic)
- Fiore, C., & Rosenblatt, L. (2023). Recapture Results and Classical Logic. *Mind*. <https://doi.org/10.1093/mind/fzad006>
- History for Clarke-Doane, Justin (forthcoming). (n.d.). [Philpapers.org](https://philarchive.org/versions/CLAWIL-3). <https://philarchive.org/versions/CLAWIL-3>
- Moschovakis, J. (2023). Intuitionistic Logic (E. N. Zalta & U. Nodelman, Eds.). *Stanford Encyclopedia of Philosophy*; Metaphysics Research Lab, Stanford University. <https://plato.stanford.edu/entries/logic-intuitionistic/#:~:text=Bishop%20and%20his%20followers%2C%20intuitionistic>
- Panu Raatikainen Intuitionistic Logic and Its Philosophy. (n.d.). <https://philarchive.org/archive/RAAILA>
- Plantin, C. (2021, October 21). Logic: an Art of Thinking, a Branch of Mathematics | Dictionnaire de l'argumentation 2021. <http://icar.cnrs.fr/dicoplantin/logic-an-art-of-thinking-a-branch-of-mathematics-e/#:~:text=Classical%20logic%20is%20based%20on>
- Rossberg, M. (2020, January 28). Logical Nihilism | University of Connecticut Logic Group. <https://logic.uconn.edu/2020/01/28/logical-nihilism/>
- Schuman, B. F. (2021). Modality and Validity in the Logic of John Buridan. [Philarchive.org](https://philarchive.org/rec/SCHMAV-11). <https://philarchive.org/rec/SCHMAV-11>
- Shapiro, S., & Kouri Kissel, T. (2018). Classical Logic (E. N. Zalta, Ed.). *Stanford Encyclopedia of Philosophy*; Metaphysics Research Lab, Stanford University. <https://plato.stanford.edu/entries/logic-classical/>