

Modeling the Impact of Disease Transmission and Treatment Timing on Honey Bee Population and Honey Production

By Eric Shen

Author Biography

Eric Shen is currently a senior at Ridge High School in Basking Ridge, New Jersey. He has a strong passion for mathematics and computer science. After starting beekeeping, he soon realized the potential to apply his skills in these areas and started working on ways to improve the effectiveness of existing treatments of honey bees to improve their survival.

Abstract

Numerous complex dynamics exist within honey bee colonies regarding their population and honey production. Honey bees are essential for pollination and food security, but their populations are declining due to various factors such as disease, parasites, and environment stresses. While there are numerous models for bee population and disease dynamics, few models also consider honey production. This study presents a mathematical model that integrates honey bee population dynamics, disease transmission, and honey production into a single comprehensive framework. Building off existing models, this model simulates the interactions between susceptible and infected hive and forager bees, along with honey resources. The results demonstrate how disease impacts the population of bees and the honey they produce. This model simulates the active and winter seasons, accounting for disease transmission and the effects of various types of treatment. Results show that early treatment leads to higher bee populations, while late treatment is more effective in eradicating the infection, but it also results in lower honey production.

Keywords: Honey bee, honey, population, model, disease, treatment

Introduction

Honey bees play a crucial role in both agricultural systems and natural ecosystems, serving as primary pollinators for numerous plant species and contributing to global food security. However, bee populations worldwide have been experiencing significant declines, attributed to various factors including parasites, diseases, nutritional stress, pesticides, and climate change. It is important to understand the complex dynamics of bee colonies to develop effective conservation methods and disease prevention strategies.

Mathematical modeling has emerged as a useful tool for exploring the dynamics of population and the numerous factors that influence the health of the colony and the productivity of the bees.

There are numerous models that focus on population dynamics and disease transmission, but there is a lack of models that consider the aspects of population with disease, and the simultaneous effect on food production. This study aims to address the gap by developing a model that combines population dynamics, disease transmission, and honey production.

Literature Review

The mathematical modeling of honey bee populations is a well-researched subject area, with various approaches focusing on different ways of modeling. This review goes over some of the findings from existing literature and identifies where our integrated model can contribute to the field.

Population Dynamics Models

Multiple studies have developed models to describe the basic population dynamics of honey bee colonies. Russel et al. (2013) presents a quantitative model that provides a framework for understanding colony failures in terms of the characteristics of the honey bee population dynamics. This model considers the age structure and division of labor among the bees.

Building on this foundation, Bagheri and Mirzaie (2019) introduces a mathematical model that explores the effect of pollen availability on colony failure. Their work highlights the critical role of nutrition in maintaining healthy bee populations.

Disease Transmission Models

The impact of diseases on honey bee populations has been a significant focus of modeling efforts. Khoury et al. (2011) developed a model that specifically examined the effects of infection on honey bee population dynamics. Their research identified two key factors in determining colony survival or collapse in the face of infection: the rate of disease transmission and the disease-induced death rate.

Betti et al. (2014) further explored the interactions between disease dynamics and colony health. Their model also includes a function for modeling honey production but does not analyze the effects of honey on the population, nor the effects of disease on honey production.

Integrated Models

There also are integrated models that incorporate numerous factors affecting bee populations. For example, the models VARROAPOP and BEEHAVE that incorporate multiple factors affecting bee populations, including infection of varroa mites, forage availability, and pesticide induced loss of bees.

Honey Production Models

Relatively fewer studies have explicitly modeled honey production in conjunction with population dynamics. However, recent work by Romero-Leiton et al. (2022) presented an approach to modeling honey bee colonies that included both population dynamics and honey production. Their research showed that when bee populations are subjected to stress factors (such as habitat destruction, Varroa mites, or climate variability), both the abundance of individuals and honey production decrease over time.

Methods

Description

Our model is based on Betti et al. (2014). It integrates bee population dynamics, disease transmission, and honey production into a single comprehensive framework. The model is based on a system of ordinary differential equations (ODEs)

that describe the interactions between different bee populations and food resources over time.

The model considers four distinct bee populations:

- H_S : Susceptible hive bees
- H_I : Infected hive bees
- F_S : Susceptible forager bees
- F_I : Infected forager bees

Additionally, the model tracks the amount of food (f) stored in the hive.

The key equations in our model for the active season are:

$$\frac{dH_S}{dt} = LS - H_S R - (\beta H_I + \beta F_I) H_S$$

$$\frac{dH_I}{dt} = (\beta H_I + \beta F_I) H_S + H_I R - dH \cdot H_I$$

$$\frac{dF_S}{dt} = H_S R - m F_S - (\beta H_I + \beta F_I) F_S$$

$$\frac{dF_I}{dt} = (\beta H_I + \beta F_I) F_S - m F_I - dF \cdot F_I$$

$$\frac{df}{dt} = c(F_S + F_I) - \gamma N$$

Where:

- L is the egg-laying rate
- S is the egg survival rate
- R is the recruitment rate of hive bees to foragers
- β is the disease transmission rate
- m is the natural death rate of foragers
- dH and dF are disease-induced death rates for hive bees and foragers
- c is the food gathering rate per forager
- γ is the food consumption rate per bee
- N is the total bee population

The equations for the winter season

$$\frac{dH_S}{dt} = -m_w H_S - (\beta H_I + \beta F_I) H_S$$

$$\frac{dF_S}{dt} = -m_w F_S - (\beta H_I + \beta F_I) F_S$$

$$\frac{dH_I}{dt} = (\beta H_I + \beta F_I) H_S - (m_w + dH) H_I$$

$$\frac{dF_I}{dt} = (\beta H_I + \beta F_I) F_S - (m_w + dF) F_I$$

$$\frac{df}{dt} = \gamma N - \gamma LS$$

Implementation

The model was implemented using Python, and the SciPy library for numerical integration of ODEs. Additionally, a web application was developed, using Flask, to provide an interactive interface, where the functions were graphed using Chart.js, to adjust parameters and visualize results. Every variable involved in the equation can be modified from the application. The code is located at <https://github.com/erics118/bee-pop-modeling>.

Parameter Values

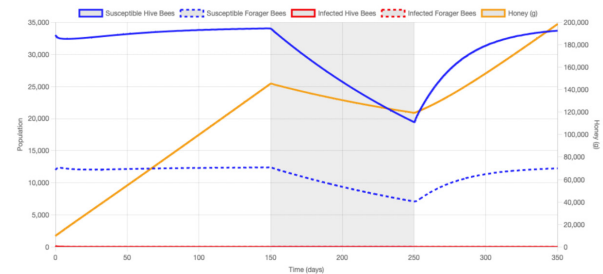
Parameters for the model were based on the existing literature. Mainly from Betti.

The parameters related to disease were varied to explore different situations of disease impact on the colony.

Results

No Infection Rate

Figure 1
No Infection Rate



This graph illustrates the baseline conditions of the honeybee population, with no infection present, and no food collection ($\beta=0$).

In the first 150 days, this is considered the active season, where forager bees are active and thus the beehive can produce honey. There are minor fluctuations in the population before the hive and forager ratio of bees settles to a level consistent with the model. During this time, the amount of honey stored slowly increases, as the forager bees collect honey, and all the bees consume the honey at a much

slower rate. This occurs because honey bees purposely store extra honey as a safety measure. They would produce less honey when there is already a huge surplus of honey, which isn't considered by the model.

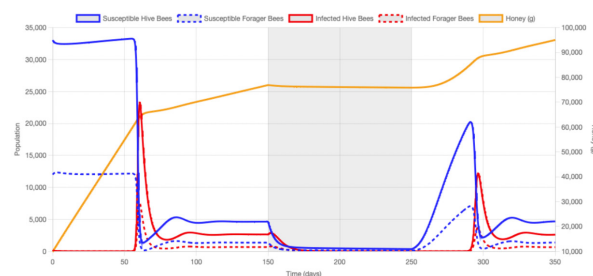
Over the winter season, the forager bees are not active, so there is a significant decrease in the honey, as the bees continue to consume honey without them producing more honey. Additionally, the population of bees decreases dramatically because there is a much lower maintenance population required for the hive over the winter season, as described by the winter season governing equations.

Once winter begins, the hive bee population decreases but recovers after winter ends, returning to a pre-winter level.

Forager bee population remains stable throughout the active season, decreasing slightly over the winter, and quickly rebounds once winter ends.

With disease

Figure 2
Low Death Rate

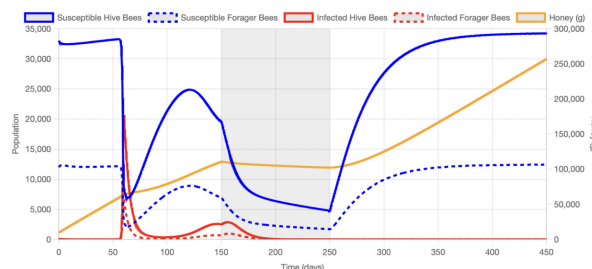


This graph represents the bee population dynamics under conditions of low death rate. On day 50, the model starts considering infection rate and death rate as factors, causing the population to plummet significantly. The infected population significantly increases, while the susceptible population decreases dramatically. By around day 100, the model stabilizes itself at the new equilibrium, until the winter season, which starts at day 150.

Then, the population remains extremely low, but survives the winter. The population then rebounds back after the winter, back to the pre-winter, but post-infection levels.

Treatment on day 60

Figure 3
Low Death Rate and Treatment on Day 60



The graph is extended to a total of 450 days to show that the model is stable at this point, showing that the model does indeed return to pre-infection population levels.

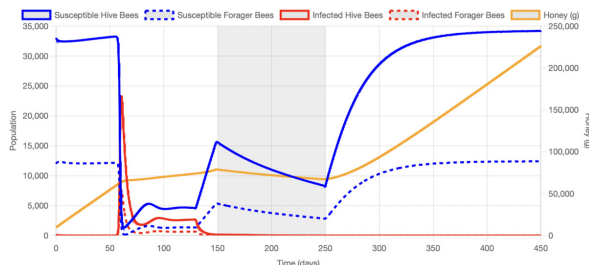
The first 50 days show a population consistent with no infection rate, as infection rate is set to 0. On day 50, the infection rate is adjusted to be the standard value, and on day 60, “treatment” is started. Our definition of treatment is setting the transmission rate to 0.00001. The population quickly seems to follow a relatively smooth trajectory, with the population of susceptible hive and forager bees increasing.

By the time winter season starts, the bee population seems to have recovered a lot. However, just as the winter season is about to start, the infected bee population spikes, which quickly is eliminated, as the total number of bees starts to decrease due to the effects of the winter season.

Then, after the winter season ends, the population quickly rebounds to the pre-infection season rates, but then dies down again due to the still present infection rate. The population of bees drops dramatically starting on day 60 and starts recovering rapidly immediately after. The population of bees peaks at day 138 or so, before going into the winter. Over the course of the winter, the population declines rapidly, and then more steadily, before rebounding back to around 60% of the original.

Treatment on day 130

Figure 4
Low Death Rate and Treatment on Day 130



This model is the exact same as the “with disease” model from days 0-130. On day 130, treatment starts. This population has a small but sharp increase in population while infection rate goes down from days 130-150 right before winter starts.

On day 130, the population starts to rise dramatically, as the hive has recovered completely from the infection. The rest of the model parallels the “no infection rate” model. This model is the best-case scenario in terms of treatment.

Discussion

There are various differences in an earlier treatment versus a later treatment. Treating the infection at day 60, just as the infection has started to take over, allows for the bees to rebuild the hive back to near pre-infection levels. This also allows for a higher honey production by the end of the season, bee population is higher throughout the season. However, this also means the likelihood for residual infected bees to survive and continue to spread disease under certain circumstances is much greater.

Treating the infection at day 130, right before the winter season, has its own set of benefits. The bee population is significantly lower throughout the duration of the active season. However, the infected bees are eradicated as the winter season starts.

The standard practice is to treat the hive later in the season. This is intended so that the infected bees do not transmit the sickness to the other bees that will be living in the hive over the winter season. Late treatment is timed to minimize this risk by targeting

a point when the infection is at its peak, eradicating it before winter begins. With such a late treatment, the infection can essentially be eradicated as winter approaches.

A downside of such a late treatment is that less honey is produced over the active season. In the model, a difference of approximately 30,000 grams of honey exists by day 450.

However, a late treatment does mean that there may need to be other treatments earlier in the season to manage the infection, before hopefully eradicating the infection fully just before the winter season starts.

Limitations

This model fails to consider many factors that will greatly affect the results. Environmental stresses such as weather, pollen, flowers, and local human activity are not considered. Additionally, factors relating to the queen bee, which is the sole producer of brood in the hive, are not considered in this model. Although this model does provide results that support standard practice in beekeeping, it still is important to realize that this model is an extreme oversimplification of reality.

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